6. L. E. Dunbar, J. F. Courtney, and L. D. McMillen, "Heating augmentation in erosive hypersonic environments," AIAA J., 13, No. 7 (1975).
7. N. N. Yanenko, R. I. Solukhin, A. N. Napyrin, and V. M. Fomin, Supersonic Two-Phase Flow with Particles Not in Velocity Equilibrium [in Russian], Nauka, Novosibirsk (1980).
8. V. F. Klimkin, A. N. Papyrin, and R. I. Soloukhin, Optical Methods for Recording High-Speed Processes [in Russian], Nauka, Novosibirsk (1980).
9. S. Sou, Hydrodynamics of Multiphase Systems [Russian translation], Mir, Moscow (1971).
10. A. P. Alkhimov, A. N. Papyrin, et al., "Theoretical and experimental investigation of the motion of a gas mixture with solid particles in a Laval nozzle," Chisl. Metody. Mekh. Sil. Sredy, 9, No. 2 (1978).
11. V. I. Blagosklonov, V. M. Kuznetsov, et al., "The interaction of hypersonic multiphase flows," Zh. Prikl. Mekh. Tekh. Fiz., No. 9 (1979).
12. L. G. Loitsyanskii, Mechanics of Liquids and Gases [Russian translation], Nauka, Moscow (1970).
13. N. F. Krasnov, Aerodynamics of Bodies of Revolution [in Russian], Mashinostroenie, Moscow (1964).
14. V. S. Khlebnikov, "Readjustment of the flow between a pair of bodies of which one is located in the wake of the other in supersonic flow," Uch. Zap. Tsentr. Aero. Gidr. Inst., 7, No. 3 (1976).
15. P. Chen, Control of Flow Separation [Russian translation], Mir, Moscow (1979).

THE QUASI-TWO-DIMENSIONAL APPROXIMATION IN THE PROBLEM OF STATIONARY SUBSONIC FLOW OVER A THREE-DIMENSIONAL ANNULAR BLADE ROW

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Subsonic flows in axial compressors are usually studied on the basis of a plane or axisymmetric theory of blade rows. Results obtained using these approximations do not give an adequate description of three-dimensional flow, however. In a number of reports [1-3] attempts are made to determine the corrections to these approximate theories allowing for the three-dimensional character of the flow under certain limiting assumptions. Irrotational flow over an annular blade row was analyzed in [1], the inverse problem of blade-row theory was solved in [2], and in [3] a two-dimensional approximation was obtained from a three-dimensional theory based on the use of an acceleration potential. Nevertheless, there is no clear procedure for estimating the relative contributions of the effects which are not taken into account by the plane theory. In particular, allowance for the variation of the Mach number over the height of a blade can prove important for modern compressors.

In the present report a two-dimensional approximation is obtained from the three-dimensional theory of a nonbearing surface in the limiting case of an infinitely large number of blades and a hub ratio close to one.

1. We shall consider the subsonic adiabatic flow of an ideal gas through one blade row rotating with a constant angular velocity ω in an infinite channel between two coaxial cylinders (see Fig. 1). The absolute motion of the gas near this annular blade row will be taken as potential. We introduce a moving coordinate system Oxyz, which rotates about the axis of the cylinders with an angular velocity ω and the x axis of which coincides with the axis of rotation. In this coordinate system the Cauchy–Lagrange integral can be written in the form [4]

$$\frac{\partial'\varphi}{\partial t}+\frac{1}{2}\left[\left(\frac{\partial\varphi}{\partial x}\right)^2+\left(\frac{\partial\varphi}{\partial y}\right)^2+\left(\frac{\partial\varphi}{\partial z}\right)^2\right]-\omega z\frac{\partial\varphi}{\partial y}+\omega y\frac{\partial\varphi}{\partial z}+P=F(t), \qquad (1.1)$$

where φ is the velocity potential of the absolute motion of the gas; a prime denotes time differentiation in the moving coordinate system; F(t) is an arbitrary function of time;

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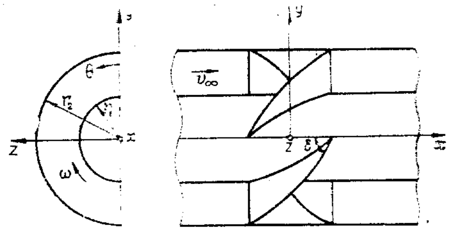

Fig. 1

$$P = \frac{\varkappa}{\varkappa - 1}\left(\frac{p_\infty^{1/\varkappa}}{\rho_\infty} p^{\frac{\varkappa-1}{\varkappa}} - \frac{p_\infty}{\rho_\infty}\right);$$

$\varkappa$, adiabatic index; p, pressure; p_∞ and ρ_∞, gas pressure and density in the undisturbed stream. In this case the continuity equation has the form

$$\partial'\rho/\partial t - \omega z \partial\rho/\partial y + \omega y \partial\rho/\partial z + \mathrm{div}\,(\rho \nabla \varphi) = 0. \tag{1.2}$$

We shall seek the solution of Eqs. (1.) and (1.2) in the form $\rho = \rho_\infty + \rho_1$, $p = p_\infty + p_1$, $v_x = v_\infty + v_{1x}$, $v_y = v_{1y}$, $v_z = v_{1z}$, where $\mathbf{v} = \nabla\varphi$, v_∞ is the axial velocity of the undisturbed stream, and the quantities ρ_1, p_1, and $\mathbf{v}_1$ are small compared with the parameters of the undisturbed stream.

We introduce a cylindrical coordinate system x, r, θ in which y = r cos θ and z = r sin θ. Then the continuity equation and the Cauchy–Lagrange integral for the disturbed gas motion take the form

$$\frac{\partial'\rho_1}{\partial t} + \omega\frac{\partial\rho_1}{\partial\theta} + \rho_\infty\left(\frac{\partial v_{1r}}{\partial r} + \frac{1}{r}v_{1r} + \frac{1}{r}\frac{\partial v_{1\theta}}{\partial\theta} + \frac{\partial v_{1x}}{\partial x}\right) + v_\infty\frac{\partial\rho_1}{\partial x} = 0; \tag{1.3}$$

$$\frac{p_1}{\rho_\infty} = -\frac{\partial'\varphi_1}{\partial t} - v_\infty\frac{\partial\varphi_1}{\partial x} - \omega\frac{\partial\varphi_1}{\partial\theta}. \tag{1.4}$$

Linearization of the adiabatic equation gives the relation $p_1 = a_\infty^2\rho_1$, where a_∞ is the speed of sound in the undisturbed stream. Substituting this expression and Eq. (1.4) into Eq. (1.3), we obtain

$$\frac{\partial^2\varphi_1}{\partial r^2} + \frac{1}{r}\frac{\partial\varphi_1}{\partial r} + \left(\frac{1}{r^2} - \frac{\omega^2}{a_\infty^2}\right)\frac{\partial^2\varphi_1}{\partial\theta^2} - 2\frac{v_\infty\omega}{a_\infty^2}\frac{\partial^2\varphi_1}{\partial x\partial\theta} + \left(1 - \frac{v_\infty^2}{a_\infty^2}\right)\frac{\partial^2\varphi_1}{\partial x^2} - 2\frac{\omega}{a_\infty^2}\frac{\partial^2\varphi_1}{\partial t\partial\theta} - 2\frac{v_\infty}{a_\infty^2}\frac{\partial^2\varphi_1}{\partial t\partial x} - \frac{1}{a_\infty^2}\frac{\partial^2\varphi_1}{\partial t^2} = 0. \tag{1.5}$$

Thus, the investigation of the disturbed potential gas flow near a three-dimensional annular blade row comes down to the solution of the linear differential equation (1.5).

2. Now let us assume that the gas flow is established, and perform a stretching transformation along the x coordinate ($x \to x/\beta$). Then, dropping the index 1, from Eq. (1.5) we obtain

$$\Delta\varphi = \sigma\left(2\frac{\partial^2\varphi}{\partial x\partial\theta} + \frac{\omega\beta}{v_\infty}\frac{\partial^2\varphi}{\partial\theta^2}\right), \tag{2.1}$$

where $\sigma = M^2\omega/v_\infty\beta$; $\beta^2 = 1 - M^2$; $M = v_\infty/a_\infty$ is the axial Mach number. Equation (2.1) can also be written in the form

$$\mathrm{div}\,\mathbf{U} = 0, \tag{2.2}$$

if we introduce the vector U with the components

$$U_x = \frac{\partial\varphi}{\partial x} - \sigma\frac{\partial\varphi}{\partial\theta},\quad U_r = \frac{\partial\varphi}{\partial r},\quad U_\theta = -\sigma r\frac{\partial\varphi}{\partial x} + \frac{1}{r}\left(1 - \frac{\omega\sigma\beta}{v_\infty}r^2\right)\frac{\partial\varphi}{\partial\theta}. \tag{2.3}$$

A green's equation can be obtained for a function φ satisfying Eq. (2.2) in a certain region Ω. For this we introduce a certain function f having continuous second derivatives and we construct a vector W from Eqs. (2.3) in which we replace φ by f. We integrate the difference

$$f \operatorname{div} \mathbf{U} - \varphi \operatorname{div} \mathbf{W} = \operatorname{div}(f\mathbf{U}) - \operatorname{div}(\varphi \mathbf{W})$$

over the volume Ω. Using the Gauss—Ostrogradskii equation, we obtain

$$\int_\Omega (f \operatorname{div} \mathbf{U} - \varphi \operatorname{div} \mathbf{W})\, d\Omega = \int_S (f\mathbf{U} - \varphi \mathbf{W})\, \mathbf{n}\, dS,$$

where S is the surface bounding the volume Ω; $\mathbf{n} = (n_x, n_r, n_\theta)$ is the outward normal to this surface.

Now let $f = G(\mathbf{x} - \mathbf{y})$ be the fundamental solution of Eq. (2.1). Here $\mathbf{x} = (x, r, \theta)$ and $\mathbf{y} = (\xi, \rho, \psi)$. Then for points $\mathbf{x} \in \Omega$ we have the representation

$$\varphi = \varphi_0 + \sigma(\varphi_1 - \varphi_2), \tag{2.4}$$

where $\varphi_0 = \int_S \left(G \frac{\partial \varphi}{\partial n} - \varphi \frac{\partial G}{\partial n}\right) dS$;

$$\varphi_1 = \int_S \varphi \left[\frac{\partial G}{\partial \psi} n_\xi + \rho \left(\frac{\partial G}{\partial \xi} + \frac{\omega \beta}{v_\infty} \frac{\partial G}{\partial \psi}\right) n_\psi\right] dS;$$

$$\varphi_2 = \int_S G \left[\frac{\partial \varphi}{\partial \psi} n_\xi + \rho \left(\frac{\partial \varphi}{\partial \xi} + \frac{\omega \beta}{v_\infty} \frac{\partial \varphi}{\partial \psi}\right) n_\psi\right] dS.$$

Next, we calculate the velocity vector $\mathbf{v} = \nabla \varphi$ using the Green's equation (2.4). We have

$$\nabla \varphi_0 = \int_S [\nabla_x G (\mathbf{n} \cdot \nabla_y) \varphi - \varphi (\mathbf{n} \cdot \nabla_y) \nabla_x G]\, dS, \quad \mathbf{y} \in S. \tag{2.5}$$

Taking into account the relations

$$(\mathbf{n} \cdot \nabla_y)(\varphi \nabla_x G) = \varphi (\mathbf{n} \cdot \nabla_y) \nabla_x G + \nabla_x G (\mathbf{n} \cdot \nabla_y \varphi),$$
$$\mathbf{n} \times (\nabla_x G \times \nabla_y \varphi) = \nabla_x G (\mathbf{n} \cdot \nabla_y \varphi) - \nabla_y \varphi (\mathbf{n} \cdot \nabla_x G),$$
$$\nabla_y \varphi \times (\mathbf{n} \times \nabla_x G) = \mathbf{n}(\nabla_y \varphi \cdot \nabla_x G) - \nabla_x G (\mathbf{n} \cdot \nabla_y \varphi),$$

we write

$$\nabla_x G (\mathbf{n} \cdot \nabla_y \varphi) - \varphi (\mathbf{n} \cdot \nabla_y) \nabla_x G = -(\mathbf{n} \cdot \nabla_y)(\varphi \nabla_x G)$$
$$+ \mathbf{n} \times (\nabla_x G \times \nabla_y \varphi) - \nabla_y \varphi \times (\mathbf{n} \times \nabla_x G) + \nabla_y \varphi (\mathbf{n} \cdot \nabla_x G) + \mathbf{n}(\nabla_y \varphi \cdot \nabla_x G). \tag{2.6}$$

By virtue of the equalities $\nabla_x G = -\nabla_y G$ and $\operatorname{rot}(\nabla G) = 0$

$$\mathbf{n} \times \operatorname{rot}_y (\varphi \nabla_x G) = \mathbf{n} \times (\nabla_y \varphi \times \nabla_x G). \tag{2.7}$$

Since

$$\mathbf{n} \operatorname{div}_y (\varphi \nabla_x G) = \mathbf{n} [\nabla_x G \cdot \nabla_y \varphi + \varphi \operatorname{div}_y (\nabla_x G)],$$

we have

$$\mathbf{n}(\nabla_x G \cdot \nabla_y \varphi) = \mathbf{n} \operatorname{div}_y (\varphi \nabla_x G) - \varphi \mathbf{n} \operatorname{div}_y (\nabla_x G). \tag{2.8}$$

As a consequence of the relations $\Delta_y(\varphi \nabla_x G) = \nabla_y (\operatorname{div}_y (\varphi \nabla_x G)) - \operatorname{rot}_y \operatorname{rot}_y (\varphi \nabla_x G)$ $(\mathbf{v} \times \mathbf{n}) \times \nabla G - \nabla G (\mathbf{v} \cdot \mathbf{n}) = -\mathbf{v}(\mathbf{n} \cdot \nabla G) - \mathbf{v} \times (\nabla G \times \mathbf{n})$, from (2.5)-(2.8), after applying the Gauss—Ostrogradskii equation, we get

$$\nabla \varphi_0 = -\int_S [(\mathbf{n} \times \mathbf{v}) \times \nabla_y G + (\mathbf{n} \cdot \mathbf{v}) \nabla_y G - \varphi \mathbf{n} \Delta_y G]\, dS.$$

For the function $\nabla \varphi_1$ we have

$$\nabla \varphi_1 = \int_S \varphi \left[\frac{\partial \nabla_x G}{\partial \psi} n_\xi + \rho \left(\frac{\partial \nabla_x G}{\partial \xi} + \frac{\omega \beta}{v_\infty} \frac{\partial \nabla_x G}{\partial \psi}\right) n_\psi\right] dS.$$

Applying the Gauss—Ostrogradskii equation with allowance for the relation $\nabla_x G = -\nabla_y G$ and returning to integration over the surface S, we find

$$\nabla\varphi_2 = -\int_S \mathbf{n}\left[\frac{\partial}{\partial\xi}\left(\varphi\frac{\partial G}{\partial\psi}\right) + \frac{\partial}{\partial\psi}\left(\varphi\frac{\partial G}{\partial\xi} + \varphi\frac{\omega\beta}{v_\infty}\frac{\partial G}{\partial\psi}\right)\right] dS + \int_S \nabla\varphi\left[\frac{\partial G}{\partial\psi}n_\xi + \rho\left(\frac{\partial G}{\partial\xi} + \frac{\omega\beta}{v_\infty}\frac{\partial G}{\partial\psi}\right)n_\psi\right] dS.$$

Then calculating $\nabla\varphi_2$ and using Eq. (2.1) we arrive at the equation sought

$$\mathbf{v}(\mathbf{x}) = -\int_S [(\mathbf{n}\times\mathbf{v})\times\nabla_y G + (\mathbf{n}\cdot\mathbf{v})\nabla_y G]\, dS - \sigma\int_S \mathbf{n}\left[v_\xi\frac{\partial G}{\partial\psi} + \rho v_\psi\left(\frac{\partial G}{\partial\xi}\right.\right. \tag{2.9}$$

$$\left.\left.+\frac{\omega\beta}{v_\infty}\frac{\partial G}{\partial\psi}\right)\right] dS + \sigma\int_S \mathbf{v}\left[\frac{\partial G}{\partial\psi}n_\xi + \rho\left(\frac{\partial G}{\partial\xi} + \frac{\omega\beta}{v_\infty}\frac{\partial G}{\partial\psi}\right)n_\psi\right] dS + \sigma\int_S \rho\nabla_y G\left[v_\psi n_\xi\right.$$

$$\left.+\left(v_\xi + \frac{\omega\beta\rho}{v_\infty}v_\psi\right)n_\psi\right] dS,$$

where v_ξ and v_ψ are the components of the velocity vector **v** in the cylindrical coordinate system at the point **y**.

The integral representation (2.9) was obtained for an arbitrary point **x** lying inside the region Ω. We note that in the case of the problem of flow over a thin body it retains the same form if the integration is carried out over one side of the surface of this body, while the vector **v** is replaced by its jump [**v**] in the transition through this surface and one uses the fact that $[\mathbf{v}\cdot\mathbf{n}] = 0$. Here it is convenient to introduce the vector intensity $\gamma = \mathbf{n} \times [\mathbf{v}]$ of the vortex sheet (the surface curl). Then for M = 0 and $G = 1/|\mathbf{x} - \mathbf{y}|$ we obtain from (2.9) the Biot—Savart equation, widely used in the theory of wings and blade rows. A representation in the form (2.9) for this particular case is obtained with the help of Green's equation in [5].

3. Let us apply the results obtained to the case when S is a blade of an annular blade row and consists of a part of a helical surface defined by the equations

$$x = \theta/\omega_*\beta,\ y = r\cos\theta,\ z = r\sin\theta,\ r_1 \leqslant r \leqslant r_2,\ -\theta_0 \leqslant \theta \leqslant \theta_0, \tag{3.1}$$

where $\omega_* = \omega c/v_\infty$; c is the half-chord of the blade in projection onto the x axis; r_1 and r_2 are the radii of the inner and outer cylinders normalized to c; θ_0 is the angular coordinate of the trailing edge of the blade. The direction cosines of the normal to the surface are

$$n_x = r\omega_*\beta/d,\ n_y = \sin\theta/d,\ n_z = -\cos\theta/d$$

and an element of area is $dS = d\,d\xi\,d\rho$ where $d^2 = 1 + r^2\omega_*^2\beta^2$

Let us consider the limiting case when the number of blades is $N \to \infty$, $r_2 \to \infty$, and the hub ratio $h = r_1/r_2$ tends toward unity. We can show that in this case Eq. (2.9) describes a certain two-dimensional gas flow through the blade row. From physical considerations it is clear that the velocity field in the radial direction does not vary as $h \to 1$. Therefore, the vector γ had only a radial component γ_r and free vortices coming off into the wake behind the blade are absent. Then for the axial and circular velocity components we obtain from (2.9)

$$v_x = \int_{-1}^{1}\int_{r_1}^{r_2} \gamma_r d\left[\left(M^2\omega_*^2\rho - \frac{1}{\rho}\right)\frac{\partial G}{\partial\psi} + \frac{M^2}{\beta}\omega_*\rho\left(1 + \frac{1}{d^2}\right)\frac{\partial G}{\partial\xi}\right] d\rho\, d\xi; \tag{3.2}$$

$$v_\theta = \int_{-1}^{1}\int_{r_1}^{r_2} \gamma_r\left[d\frac{\partial G}{\partial\xi}\cos(\theta-\psi) + \frac{M^2\omega_*}{\beta}\left(\frac{\rho}{rd} - d\cos(\theta-\psi)\right)\frac{\partial G}{\partial\psi}\right] d\rho\, d\xi. \tag{3.3}$$

The fundamental solution of Eq. (2.1) for the region between the cylinders $r = r_i$ (i = 1, 2) has the form [3]

$$G(\mathbf{x}-\mathbf{y}) = \frac{Nr_2}{2\pi}\mathrm{Re}\sum_{n,m=1}^{\infty}\frac{1}{\Omega_{km}}R_k\left(\lambda_{km}\frac{r}{r_2}\right)R_k\left(\lambda_{km}\frac{\rho}{r_2}\right)E_{km}(\mathbf{x},\mathbf{y}) \tag{3.4}$$

$$-G_0 + \frac{Nr_2}{4\pi}\sum_{m=1}^{\infty}\frac{1}{\lambda_{0m}}R_0\left(\lambda_{0m}\frac{r}{r_2}\right)R_0\left(\lambda_{0m}\frac{\rho}{r_2}\right)e^{-\frac{\lambda_{0m}}{r_2}|x-\xi|},$$

where

$$E_{km}(\mathbf{x}, \mathbf{y}) = e^{ik(\theta-\psi) + \frac{i}{\beta} kM^2\omega_*(x-\xi) - \frac{\Omega km|x-\xi|}{r_2}}; \; G_0 = \frac{N}{2\pi r_2(1-h^2)}(|x-\xi| +$$

$$+ x - \xi); \; \Omega_{km}^2 = \lambda_{km}^2 - k^2M^2\omega_*^2 r_2^2/\beta^2 \; (k = nN); \; \lambda_{km} -$$

λ_{km} are the roots of the system of equations $R_k'(\lambda) = 0$, $R_k'(\lambda h) = 0$, $R_k(r) = A_k\,[J_k(r)Y_k'(\lambda h) - J_k'(\lambda h)Y_k(r)]$; A_k is a normalizing factor; a prime denotes differentiation; J_k and Y_k are first- and second-order Bessel functions, respectively.

In the case under consideration $k \gg 1$, so that [6] $\lambda_{km} \sim k$ regardless of m and $\Omega_{km} = k\beta_2/\beta$. Here $\beta_2^2 = 1 - M_2^2$ and M_2 is the Mach number at the end of the blade ($r = r_2$). Consequently, the functions E_{km} do not depend on the index m, and it will be omitted from now on. Since the blades differ little from stream surfaces of the undisturbed motion, the stagger $\delta = \delta(r)$ of the blade row in a cross section r = const can be determined from the relation

$$\operatorname{tg}\delta = \beta\omega_* r. \tag{3.5}$$

Substituting (3.4) and (3.5) into (3.2) and introducing the notation

$$q_{km} = \int_{r_1}^{r_2} \frac{\gamma_r}{\rho\cos\delta}\left(\frac{M^2}{\beta^2}\operatorname{tg}^2\delta - 1\right) R_k\left(\lambda_{km}\frac{\rho}{r_2}\right) d\rho,$$

$$p_{km} = \int_{r_1}^{r_2} \frac{\gamma_r}{\cos\delta}\operatorname{tg}\delta\,(1+\cos^2\delta)\, R_k\left(\lambda_{km}\frac{\rho}{r_2}\right) d\rho,$$

we find

$$v_x = \frac{\beta r_2 N}{2\beta_2}\int_{-1}^{1}\sum_{n,m=1}^{\infty} q_{km}R_k\left(\lambda_{km}\frac{r}{r_2}\right)\operatorname{Im}E_k(\mathbf{x},\mathbf{y})\,d\xi - \tag{3.6}$$

$$-\frac{M^2N}{2\pi\beta\beta_2}\int_{-1}^{1}\sum_{n,m=1}^{\infty} p_{km}R_k\left(\lambda_{km}\frac{r}{r_2}\right)\operatorname{Re}\left\{\left[\frac{i}{\beta}M^2\omega_* r_2 + \frac{\beta_2}{\beta}\operatorname{sign}(\xi - x)\right]E_k(\mathbf{x},\mathbf{y})\right\}d\xi - \frac{M^2N}{4\pi\beta^2}$$

$$\times\int_{-1}^{1}\left\{\sum_{m=1}^{\infty} p_{0m}R_0\left(\lambda_{0m}\frac{r}{r_2}\right)\operatorname{sign}(\xi - x)\,e^{-\frac{\lambda_{0m}}{r_2}|x-\xi|} + p_{00}[\operatorname{sign}(\xi - x) - 1]\right\}d\xi \quad \left(\lambda_{00} = 0,\; R_0^2(0) = \frac{2}{r_2^2(1-h^2)}\right).$$

To calculate the aerodynamic characteristics of blade rows it is sufficient to know the velocity field near a blade. In this case the value of $|x - \xi|$ is finite, while $\lambda_{0m} \to \infty$ as $h \to 1$, so that the penultimate term in (3.6) tends toward zero.

In connection with the fact that q_{km} and p_{km} can be treated as coefficients of expansions of the functions

$$\frac{\gamma_r}{\rho^2\cos\delta}\left(\frac{M^2}{\beta^2}\operatorname{tg}^2\delta - 1\right) \text{ and } \frac{\gamma_r}{\rho\cos\delta}(1+\cos^2\delta)\operatorname{tg}\delta,$$

in Fourier–Bessel series, we can write

$$v_x = \frac{\beta r_2 N}{2\pi\beta_2 r^2\cos\delta}\left(\frac{M^2}{\beta^2}\operatorname{tg}^2\delta - 1\right)\int_{-1}^{1}\gamma_r(\xi, r)\sum_{n=1}^{\infty}\operatorname{Im}E_k\,d\xi -$$

$$-\frac{M^2N}{2\pi\beta\beta_2}\frac{(1+\cos^2\delta)\operatorname{tg}\delta}{r\cos\delta}\int_{-1}^{1}\gamma_r(\xi, r)\left\{\sum_{n=1}^{\infty}\operatorname{Re}\left[\left(\frac{i}{\beta}M^2\omega_* r_2 + \frac{\beta_2}{\beta}\operatorname{sign}(\xi - x)\right)E_k\right] + \frac{\beta_2}{2\beta}[\operatorname{sign}(\xi - x) - 1]\right\}d\xi.$$

Let us assume that in the limiting transition the quantity $\tau = N/2\pi r_2$ remains constant while the angle δ approaches its value at $r = r_2$. We introduce the new variable $y = r_2\theta$ and the notation

$$X = (2\pi\beta_2\tau/\beta)(\xi - x), \; Y = 2\pi\tau\,[y + (1/\beta^2)(M^2x - \xi)\operatorname{tg}\delta].$$

Then, if we use the expressions [7]

$$\sum_{n=1}^{\infty} e^{-nx} \sin n\theta = \frac{1}{2} \frac{\sin\theta}{\operatorname{ch} x - \cos\theta}, \quad \sum_{n=1}^{\infty} e^{-nx} \cos n\theta = \frac{1}{2}\left(\frac{\operatorname{sh} x}{\operatorname{ch} x - \cos\theta} - 1\right)$$

for x > 0, we can represent Eq. (3.2) in the form

$$v_x = \frac{\beta\tau}{2\beta_2}\left[\frac{M^2}{\beta^4}(1 + M^2\cos^2\delta)\ \operatorname{tg}^2\delta - 1\right]\int_{-1}^{1} \frac{\gamma_r}{\cos\delta} \frac{\sin Y}{\operatorname{ch} X - \cos Y} d\xi$$

$$- \frac{M^2\tau'}{2\beta^2}(1 + \cos^2\delta) \operatorname{tg}\delta \int_{-1}^{1} \frac{\gamma_r}{\cos\delta}\left(\frac{\operatorname{sh} X}{\operatorname{ch} X - \cos Y} - 1\right) d\xi. \qquad (3.7)$$

Using an analogous procedure, we find

$$v_y = v_\theta = -\frac{\tau}{2}\int_{-1}^{1} \frac{\gamma_r}{\cos\delta}\left(\frac{\operatorname{sh} X}{\operatorname{ch} X - \cos Y} - 1\right) d\xi + \frac{M^2\tau\sin 2\delta}{4\beta\beta_2}\int_{-1}^{1} \frac{\gamma_r}{\cos\delta} \frac{\sin Y}{\operatorname{ch} X - \cos Y} d\xi. \qquad (3.8)$$

Equations (3.7) and (3.8) for M = 0 do not differ from the corresponding equations given by the plane theory. Thus, in the case of an uncompressed fluid as $N \to \infty$, $r_2 \to \infty$, and $h \to 1$ the model of plane flow is a sufficiently good approximation for calculating the aerodynamic characteristics of an axial compressor.

In the case of a compressed stream additional terms containing M^2 and functions depending on the stagger of the blade row at the periphery appear, and hence one cannot obtain the equivalent flow of an incompressible fluid through the blade row using a Prandtl–Glauert transformation. This is connected with the fact that Eq. (2.1) is not reduced to the Laplace equation which would be obtained in the plane theory in the coordinate system being used.

LITERATURE CITED

1. J. E. McCune, "A three-dimensional theory of axial compressor blade rows — application in subsonic and supersonic flows," J. Aero/Space Sci., 25, No. 9 (1958).
2. O. Okurounmu and J. E. McCune, "Lifting surface theory of axial compressor blade rows: Part I — subsonic compressor," AIAA J., 12, No. 10 (1974).
3. M. Namba, "Lifting surface theory for a rotating subsonic or transonic blade row," Br. Aeronaut. Res. Coun. Rep. Memo, No. 3740 (1972).
4. N. E. Kochin, I. A. Kibel', and N. V. Roze, Theoretical Hydromechanics [in Russian], Part 1, Fizmatgiz, Moscow (1963).
5. G. M. Morgunov, "The three-dimensional analog of the generalized Cauchy equation and one of its applications in hydrodynamics," Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza, No. 2 (1974).
6. G. N. Watson, A Treatise on the Theory of Bessel Functions, 2nd edn., Cambridge Univ. Press (1944), Part 1.
7. I. S. Gradshtein and I. M. Ryzhik, Tables of Integrals, Series and Products, Academic Press (1966).